## 7.5: Special Types of Linear Systems

Goals: *Solve and identify when a system of equations has one solution, no solution or an infinite number of solutions
*Arrange systems so you can state the number of solutions without solving

The graph below is the system of equations: $x+2 y=6$ and $2 x-3 y=-2$


## What is the solution to the system? How do you know?

Solution: __(2, 2), it's the point of intersection $\qquad$
How could you prove this is the solution to the system?
By plugging the point into both equations and it should be true in both.

## **Review/Preview**

- What is a solution to a linear system?

1) An ordered pair that when substituted in works for both equations
2) A point on both lines (the point of intersection)

Now graph each system of equations using slope-intercept form or $\boldsymbol{x}$ and $\boldsymbol{y}$ intercepts. Then state the solution to each system of equations.

Ex: $y=2 x+5$
$y=2 x+1$


NO SOLUTION

Ex: $-6 x+2 y=-2$
$y=3 x+2$


NO SOLUTION


NO SOLUTION


INFINITE SOLUTIONS

What did you say when you graphed two lines and they happened to be parallel? (Hint: If a solution is where two lines intersect, where do parallel lines intersect?)

If two lines are parallel they never intersect, which means there would be no solution to the system.

What did you say when you graphed two lines and they were the same line, one on top of the other? (Hint: How many points of intersection are there?)

If two lines are the same then they have infinite points in common, which means there are infinite solutions.

1. Substitute or Eliminate depending on which method is most efficient.
2. Then graph each system of equations on the coordinate plane provided.
3. Based on your graph, state the solution to the system of equations.
4. Explain how the algebra (substitution or elimination) supports this solution?

Ex: $3 x+2 y=10$
$\begin{array}{r}3 x+2 y=2 \\ \hline 0=8\end{array}$
Wait, where did the variable go? And $0 \neq 8$ !
Look at the graph... parallel lines. NO SOLUTION!
So if $0=8$ this must mean NO SOLUTION!


Ex: $x-2 y=-4$

$$
\begin{aligned}
& y=\frac{1}{2} x+2 \\
& x-2\left(\frac{1}{2} x+2\right)=-4 \\
& x-x-4=-4 \\
& -4=-4
\end{aligned}
$$

Where did the variable go? And $4=4$ !
Look at the graph...same line. INFINITE SOLUTIONS! So if $4=4$ this must be INFINITE SOLUTIONS!


$$
\text { Ex: } \begin{aligned}
5 x+3 y & =6 \\
-5 x-3 y & =3 \\
\hline 0 & =9
\end{aligned}
$$

NO SOLUTION!


Ex: $y=2 x-4$

$$
-6 x+3 y=-12
$$

$$
-6 x+3(2 x-4)=-12
$$

$$
-6 x+6 x-12=-12
$$

$$
-12=-12
$$

INFINITE SOLUTIONS!


Solve each equation or inequality.

Ex: $3(x+4)=3 x+16$

$$
\begin{aligned}
& 3 x+12=3 x+16 \\
& 12=16 \\
& \text { No solution }
\end{aligned}
$$

Ex: $2 x-3 x+6 \leq-(x-10)$
$-x+6 \leq-x+10$
$6 \leq 10$
Any number

Ex: $4(2 x+6)=8(x+3)$
$8 x+24=8 x+24$ $24=24$
Any number
Ex: $\begin{aligned} & 3(6 x-1)>2(9 x-1) \\ & 18 x-3>18 x-2 \\ &-3>-2 \\ & \text { No Solution }\end{aligned}$
*Regardless of if you are solving an equation or an inequality what is the general rule that applies to both types of problems?

If you get a true statement then the solution is "any number"
If you get a false statement then the solution is "no solution"

Now we are going to apply this same concept to systems of equations...
Solve each system by eliminating or substituting.

Ex: $2 x-3 y=6$
$\begin{array}{r}-2 x-3 y=-4 \\ \hline 0=10\end{array}$
NO SOLUTION!

Ex: $4 x-2 y=8$
$y=2 x-4$
$4 x-2(2 x-4)=8$
$4 x-4 x+8=8$
$8=8$
INFINITE SOULTIONS

Identify the number of solutions of a linear system:

- A system of equations will have no solution when the two lines are: Parallel

They are $\qquad$ parallel $\qquad$ when: They have the same slope but different $y$-intercepts

- A system of equations will have an infinite number of solutions when the two lines are: exactly the same

They are $\qquad$ the same line $\qquad$ when: they have the same slope and $y$-intercept

- A system of equations will have exactly one solution when the two lines are: Not Parallel

They are $\qquad$ not Parallel $\qquad$ when: Their slopes are different. The $y$-intercept is irrelevant

| Number of Solutions | Slopes and $y$-intercepts |
| :---: | :---: |
| One | $m=$ different <br> $b=$ same or different |
| None | $m=$ same <br> $b=$ different |
| Infinite | $m=$ same <br> $b=$ same |

If you can quickly identify the slope and $y$-intercept of each line, then you can state how many solutions the system has without solving.

- What do you need to do to be able to quickly identify the slope and $y$-intercept of a line?

The line needs to be in slope-intercept form ( $y$ needs to be isolated!)
Without solving the system, tell whether there is one solution, no solution or infinitely many solutions.

$$
\begin{aligned}
& \text { Ex: }-2(5 x+y=-2) \rightarrow-10 x-2 y=4 \\
& -10 x-2 y=4 \\
& y=-2-5 x \\
& y=-2-5 x
\end{aligned}
$$

Infinite Solutions, same slope and $y$-intercept
OR: You can make the top equation match the bottom

$$
\text { Ex: } \begin{aligned}
& 6 x+2 y=3 \\
& 6 x+2 y=-5 \\
& y=-3 x+1.5 \\
& y=-3 x-2.5
\end{aligned}
$$

No solution, same slope different $y$-intercept!
OR: Both equations start with $6 x+2 y$ but end diff.
Ex: $-3 x+5 y=6$
$6 x-10 y=-12$

$$
\text { Ex: } \begin{aligned}
9 x-5 y & =12 \\
& 9 x-5 y
\end{aligned}=8
$$

Infinite Solutions
No solution

$$
\text { Ex: } \begin{aligned}
x-3 y & =-15 \\
2 x-3 y & =-18
\end{aligned}
$$

## One Solution

## Use the graphs below to show a system of equations with:

a. No solution
b. One solution



> c. Infinitely many solutions


