7.3: Solve Systems of Equations by Adding or Subtracting

Goals: *Find the solution to a system of equations by eliminating a variable using addition or subtraction
*Arrange systems so you can eliminate

By which two methods can you already solve a system?
Graphing and substitution

Now you will be able to solve an equation by ELIMINATING a variable!!

Ex: $2x + 3y = 11$
$+\ -2x + 5y = 13$ — If you add these two equations together then the x's will cancel out.
$8y = 24$ — Solve for y
$y = 3$ — Now that you know y, plug it into either equation and solve for x

$2x + 3(3) = 11$
$2x + 9 = 11$
$2x = 2$
$x = 1$ — Solution: (1, 3)

Ex: $4x + 3y = 2$
$5x + 3y = -2$

(–4, 6)

Ex: $3x + 4y = 8$
$-3x + 5y = 10$

(0, 2)

Ex: $5x + 6y = 4$
$7x + 6y = 8$

(2, –1)

Ex: $8x - 4y = -4$
$4y = 3x + 14$

Since these equations are not lined up, you first need to rearrange them so the variables and equals sign are lined up

$8x - 4y = -4$
$+\ -3x + 4y = 14$ — Now add.
$5x = 10$ — Solve for x
$x = 2$ — Find y
$y = 5$ — (2, 5)

Ex: $9x - 3y = 18$
$3y = -7x + 30$

(3, 3)

7.4: Solve Systems of Equations by Multiplying

Goals: *Find the solution to a system of equations by eliminating a variable using multiplication

***Can you add or subtract these equations as they written and still eliminate one of the variables?**

$5x + 2y = 16$
$3x - 4y = 20$ No- if you add you get $8x$ and $-2y$ and if you subtract you get $2x$ and $6y$

***Could you manipulate either equation so you COULD eliminate a variable?**

The golden rule of equations is "As long as you do something to one side, do it to the other." If you multiply the first equation by 2 on both sides, then the y coefficients will have the same absolute value and you can add the equations together to eliminate a variable.

$$\begin{array}{lcr} 2(5x + 2y = 16) & \rightarrow & 10x + 4y = 32 \\ 3x - 4y = 20 & \rightarrow & \underline{+\ 3x - 4y = 20} \\ & & 13x = 42 \\ & & x = 4 \\ & & y = -2 \\ & & (4, -2) \end{array}$$

Add.

Plug back into either of the two original equations. These are better because the numbers are smaller.

Ex: $6x + 5y = 19$
$2x + 3y = 5$

(4, –1)

Ex: $2x + y = -9$
$4x + 11y = 9$

(–6, 3)

Ex: $4x + 5y = 35$
$3x - 2y = 9$

(5, 3)

Ex: $3x - 7y = 5$
$9y = 5x + 5$

(–10, –5)

Ex: $2x - 3y = 6$
$4y = -7x - 8$ (0, –2)

Ex: During a kayaking trip a kayaker travels 12 miles upstream (against the current) and 12 miles downstream (with the current). It took 3 hours to go upstream and 2 hours to go downstream. The speed of the current stayed the same throughout the trip. Find the average speed of the kayaker and the average speed of the current.

x = speed of the kayaker y = speed of the current $d = rt$

When you go downstream, the current is added to your speed. So the kayaker's speed (r) would be $x + y$
The distance is 12 miles and the time is 2 hours.

$12 = 2(x + y)$

When you go upstream, the current pushes against your speed, causing it to be less. So the kayaker's speed would be $x - y$. The distance is still 12 mile and the time is 3 hours.

$12 = 3(x - y)$

Use these two equations to form a system

$12 = 2(x + y) \rightarrow 12 = 2x + 2y \rightarrow 3(12 = 2x + 2y) \rightarrow 36 = 6x + 6y$
$12 = 3(x - y) \rightarrow 12 = 3x - 3y \rightarrow 2(12 = 2x - 2y) \rightarrow \underline{24 = 6x - 6y}$ Add. (Easier than subtraction)
$60 = 12x$
$x = 5$ Find y. Plug into an original.
$y = 1$
Speed of kayaker = 5 mph, speed of current = 1 mph

Ex: A riverboat travels 28 miles upstream in 7 hours. It travels 28 miles downstream in 5 hours. Find the average speed of the riverboat and the current.

Speed of riverboat = 4.8 mph, speed of current = 0.8 mph