3.4: Solve Equations with Variables on Both Sides

Goals: *Solve basic equations with variables on both sides
*Decide if an equation has "no solution" or "any number"

Steps to Solving Multi-Step Equations

S – Simplify

D – Distribute

C – Combine

B – Balance (reverse PEMDAS)

A – Answer

Ex: $7 - 8x = 4x - 17$

$$\begin{aligned} \underline{+8x \quad +8x} \\ 7 &= 12x - 17 \\ \underline{+17 \qquad +17} \\ \frac{24}{12} &= \frac{12x}{12} \\ x &= 2 \end{aligned}$$

Ex: $13 + 5x = 2x - 8$

$$\begin{aligned} \underline{-2x \quad -2x} \\ 13 + 3x &= -8 \\ \underline{-13 \qquad -13} \\ \frac{3x}{3} &= \frac{-21}{3} \\ x &= -7 \end{aligned}$$

Ex: $9x - 5 = \frac{1}{4}(16x + 60)$

$$\begin{aligned} 9x - 5 &= 4x + 15 \\ \underline{-4x \qquad -4x} \\ 5x - 5 &= 15 \\ \underline{+5 \quad +5} \\ \frac{5x}{5} &= \frac{20}{5} \\ x &= 4 \end{aligned}$$

Ex: $3 - 4a = 5(a - 3)$

$$\begin{aligned} 3 - 4a &= 5a - 15 \\ \underline{+4a \quad +4a} \\ 3 &= 9a - 15 \\ \underline{+15 \qquad +15} \\ \frac{18}{9} &= \frac{9a}{9} \\ a &= 2 \end{aligned}$$

Ex: $24 - 3m = 5m$

$$\begin{aligned} +3m \quad & +3m \\ \frac{24}{8} &= \frac{8m}{8} \\ 3 &= m \end{aligned}$$

Ex: $20 + c = 4c - 7$

$$\begin{aligned} -c \quad & -c \\ 20 &= 3c - 7 \\ +7 \quad & +7 \\ \frac{27}{3} &= \frac{3c}{3} \\ 9 &= c \end{aligned}$$

Ex: $9 - 3k = 17 - 2k$

$$\begin{aligned} +3k \quad & +3k \\ 9 &= 17 + k \\ -17 \quad & -17 \\ -8 &= k \end{aligned}$$

Ex: $5z - 2 = 2(3z - 4)$

$$\begin{aligned} 5z - 2 &= 6z - 8 \\ -5z \quad & -5z \\ -2 &= 1z - 8 \\ +8 \quad & +8 \\ 6 &= z \end{aligned}$$

Ex: $8y - 6 = \frac{2}{3}(6y + 15)$

$$\begin{aligned} 8y - 6 &= 4y + 10 \\ -4y \quad & -4y \\ 4y - 6 &= 10 \\ +6 \quad & +6 \\ \frac{4y}{4} &= \frac{16}{4} \end{aligned}$$

Ex: To be a member of a movie club you have to pay a one-time membership fee of \$42. Then you pay an additional \$5 to go to a movie. If you are a non-member it costs \$8 to go to a movie.

a) Write an equation to show when a member and a non-member would have the same cost.

$5x + 42 = 8x$

b) Solve your equation.

$$\begin{aligned} 5x + 42 &= 8x \\ -5x \quad & -5x \\ \frac{42}{3} &= \frac{3x}{3} \\ 14 &= x \end{aligned}$$

c) Decide how many movies it would make sense to buy a membership.

After 14 movies it would make sense to be a member.

"Solve" means to: Tell me what the variable can be…

***Normally when you solve an equation you get, for example, an answer that looks like:**

$x = 5$…

This means that… x is allowed to equal 5 and nothing else

***But sometimes when you solve an equation something fishy happens….**

…Let's investigate: Try these examples and what do you notice is different about them compared to all of the other answers we have seen so far? What do you think this means?

Ex: $3x = 3(x+4)$

$$\begin{aligned} 3x &= 3x + 12 \\ \underline{-3x} &\ \underline{-3x} \\ 0 &= 12 \end{aligned}$$

No Solution

Ex: $2x + 10 = 2(x+5)$

$$\begin{aligned} 2x + 10 &= 2x + 10 \\ \underline{-2x} &\quad \underline{-2x} \\ 10 &= 10 \end{aligned}$$

Any Number

When you solve an equation one of three things will happen:

1. The variable will be isolated and you will get one solution. It will look like $x = \#$
   This means that the <u>only</u> number that x can be is the one that it equals

2. When trying to isolate the variable, it will disappear and you will end up with a false statement. This means that your answer is:
   "No solution" This means that there is no number that x could ever equal that would work.
   An example of this is $0 = 5$

3. When trying to isolate the variable, it will disappear and you will end up with a true statement. This means that your answer is:
   "Any number" This means that x can be any number and it will work if you plug it in. An example of this is: $5 = 5$

Notice in <u>ALL THREE CASES</u> you are always trying to ISOLATE THE VARIABLE!!

Ex: $9z + 12 = 9(z + 3)$

$$\begin{aligned} 9z + 12 &= 9z + 27 \\ -9z \quad & \quad -9z \\ 12 &= 27 \end{aligned}$$

No Solution

Ex: $7w + 1 = 8w + 1$

$$\begin{aligned} 7w + 1 &= 8w + 1 \\ -7w \quad & \quad -7w \\ 1 &= 1w + 1 \\ -1 \quad & \quad -1 \\ \frac{0}{1} &= \frac{1w}{1} \\ 0 &= w \end{aligned}$$

Ex: $3(2a + 2) = 2(2a + 3)$

$$\begin{aligned} 6a + 6 &= 4a + 6 \\ -4a \quad & \quad -4a \\ 2a + 6 &= 6 \\ -6 \quad & \quad -6 \\ \frac{2a}{2} &= \frac{0}{2} \\ a &= 0 \end{aligned}$$

Ex: $4(2x + 3) = 2(4x + 6)$

$$\begin{aligned} 8x + 12 &= 8x + 12 \\ -8x \quad & \quad -8x \\ 12 &= 12 \end{aligned}$$

Any number