

Chapter 5: Writing Linear Equations Study Guide (REG)

5.1: Write equations of lines given slope and y – intercept or two points

Write the equation of the line with the given information:

Ex: Slope: 0, y – intercept: $\frac{1}{2}$

$$y = \frac{1}{2}$$

*Don't forget that $0x$ can be simplified.

Ex: Passes through (0, 5) and (1, 7)

$$y = 2x + 5$$

*Find the slope first. Remember the y-intercept can be Found easily if paired with an x-coordinate of 0.

Ex: (1, -9) and (0, -11)

$$\text{Find the slope: } \frac{-11 - (-9)}{0 - 1} = \frac{-2}{-1} = 2$$

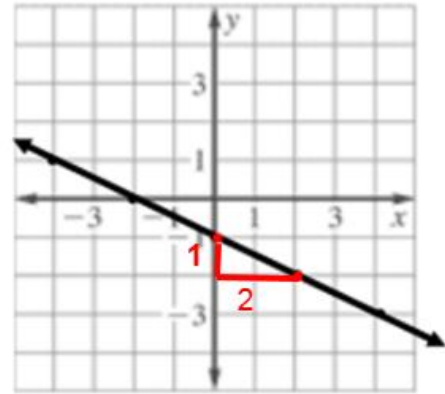
$$m = 2$$

Identify b as the number

Paired with an x-coord. of 0. $b = -11$

Write the equation. $y = 2x - 11$

Ex:



$y = -\frac{1}{2}x - 1$ *The slope is $-\frac{1}{2}$ and the y-intercept -1 .

5.2: Write the equation of lines given slope and one point, or two points

Write the equation of the line with the given information:

Ex: Slope 3, passes through (1, 1)

$$1 = 3(1) + b$$

$$1 = 3 + b$$

$$\frac{-3}{-2} = \frac{-3}{-2}$$

$$-2 = b$$

$$y = 3x - 2$$

Ex: Slope -5 , passes through $(-4, 7)$

$$7 = -5(-4) + b$$

$$7 = 20 + b$$

$$\frac{-20}{-13} = \frac{-20}{-13}$$

$$-13 = b$$

$$y = -5x - 13$$

Ex: Passes through (1, 4) (2, 7)

$$m = \frac{7-4}{2-1} = \frac{3}{1} = 3$$

$$4 = 3(1) + b$$

$$4 = 3 + b$$

$$\begin{array}{r} -3 \quad -3 \\ \hline 1 = b \end{array}$$

$$y = 3x + 1$$

Ex: Passes through (-2, -2) (1, -1)

$$m = \frac{-1-(-2)}{1-(-2)} = \frac{1}{3}$$

$$-1 = \frac{1}{3}(1) + b$$

$$-1 = \frac{1}{3} + b$$

$$\begin{array}{r} -\frac{1}{3} \quad -\frac{1}{3} \\ \hline \end{array}$$

$$-\frac{4}{3} = b$$

$$y = \frac{1}{3}x - \frac{4}{3}$$

Ex: (-3, 1) (-3, -1)

$$m = \frac{-1-1}{-3-(-3)} = \frac{-2}{0} = \text{undefined}$$

An line with an undefined slope is vertical.
Vertical line equations all start with $x =$

In this case x is always -3 so the equation is:

$$x = -3$$

Ex: (1, 5) (-7, 5)

$$m = \frac{5-5}{-7-1} = \frac{0}{-8} = 0$$

$$5 = 0(1) + b$$

$$5 = 0 + b$$

$$5 = b$$

$$y = 5$$

Ex: Passes through (9, 1) and (1, 7)

$$m = \frac{7-1}{1-9} = \frac{6}{-8} = -\frac{3}{4}$$

$$1 = -\frac{3}{4}(9) + b$$

$$1 = -\frac{27}{4} + b$$

$$+\frac{27}{4} \quad +\frac{27}{4}$$

$$\begin{array}{r} \frac{31}{4} = b \end{array}$$

$$y = -\frac{3}{4}x + \frac{31}{4}$$

Ex: (3, 1) and (6, 4)

$$m = \frac{4-1}{6-3} = \frac{3}{3} = 1$$

$$1 = 3(1) + b$$

$$1 = 3 + b$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$-2 = b$$

$$y = x - 2$$

Ex: You are taking a Tae Kwon Do class that costs \$15 a month. In addition, you needed to purchase a uniform. You paid a total of \$108 after 6 months.

- a. Find the cost of a uniform. Show or explain your work.

Identify variables first: x : # of months, y : total cost

Once x and y are identified you can create an ordered pair (6, 108)

\$15 per month is the slope since this would multiply the number of months (x) and slope always multiplies x .

$$\begin{aligned}y &= mx + b \\108 &= 15(6) + b \\108 &= 90 + b \\-90 & \quad -90 \\ \hline 18 &= b\end{aligned}$$

If you are finding the cost of a uniform, you are really finding b , since b is the y -intercept and the y -intercept always occurs when x is 0.

Since x is the number of months, if the number of months is 0 and you are still paying money, then this must be the cost of the uniform.

- b. Write an equation that gives the total cost (in dollars) as a function of the length of time you have been taking classes (in months).

$$y = 15x + 18$$

- c. Find the total cost after 9 months.

$$\begin{aligned}y &= 15(9) + 18 \\y &= 135 + 18 \\y &= 153\end{aligned}$$

Ex: You have a subscription to an online magazine that allows you to view 25 articles from the magazine's archives. You are charged an additional fee for each article after the first 25 articles viewed. After viewing 28 articles, you paid a total of \$34.80. After viewing 30 articles, you paid a total of \$40.70.

- a. What is the cost per archived article **after the first 25 articles** viewed?

IMPORTANT You pay a subscription fee, which covers the cost of the first 25 articles, then pay a fee per article only for the EXTRA articles you view.

Let x = # EXTRA articles, and y = total cost

First, find the cost per extra article by finding the slope, which means you need two ordered pairs. Since you pay \$34.80 for 28 articles, but this means only 3 of them are EXTRA the ordered pair you can create is: (3, 34.80). The second ordered pair would be (5, 40.70) since the total cost is \$40.70 for 5 EXTRA articles.

Now that you have two ordered pairs find the slope:

$$\frac{40.7 - 34.8}{5 - 3} = \frac{5.9}{2} = 2.95 \text{ is the cost per extra article.}$$

- b. What is the cost of the magazine subscription?

Now that you have the slope find the subscription fee, which is b (since this the price you would pay even if you view 0 extra articles and the y -intercept always occurs when x is 0)

$$34.8 = 2.95(3) + b$$

$$34.8 = 8.85 + b$$

$$\underline{-8.85 \quad -8.85}$$

$$25.95 = b$$

If asked the equation would have been: $y = 2.95x + 25.95$

Ex: A delivery service charges a base price for an overnight delivery of a package, plus an extra charge for each pound the package weighs. A customer is billed \$22.85 for shipping a 3-lb package and \$40 for shipping a 10-lb package.

- a. Write an equation that gives the total cost for shipping a package of any weight.

Need to find the cost per pound (slope) and the base price (y -intercept).

Let x = number of pounds and y = total cost, then make two ordered pairs:

(3, 22.85) and (10, 40)

Find the slope: $\frac{40-22.85}{10-3} = \frac{17.15}{7} = \$2.45/lb$

Now find the base price by finding b .

$$40 = 2.45(10) + b$$

$$40 = 24.5 + b$$

$$\underline{-24.5 \quad -24.5}$$

$$15.5 = b$$

$$y = 2.45x + 15.5$$

- b. Then find the cost of shipping a 15-lb package.

$$y = 2.45(15) + 15.5$$

$$y = 36.75 + 15.5$$

$$y = 52.25$$

5.5: Write Equation of Parallel and Perpendicular Lines

Ex: Write the equation of the line that is parallel to $-6x + y = -1$ and passes through the point $(1, 7)$

First find the slope of the line represented by $-6x + y = -1$ by putting it in slope-intercept form. This is the Green Line.

$$\begin{array}{r} -6x + y = -1 \\ +6x \quad \quad +6x \\ \hline y = -1 + 6x \rightarrow \text{The slope is 6.} \end{array}$$

Since the slope of the green line is 6 and the green line and blue line (that doesn't have an equation yet) are parallel, this means they have the same slope, so the blue line also has a slope of 6, and we know that it passes through the point $(1, 7)$, so we now have enough information to replace m , x , and y , in:

$$\begin{array}{l} y = mx + b \\ 7 = 6(1) + b \\ 7 = 6 + b \\ 1 = b \end{array} \quad \text{The blue has its own y-intercept of 1. So the equation is:}$$
$$y = 6x + 1$$

Ex: Write the equation of the line that is perpendicular to $y + 3 = 2x$ and passes through the point $(-5, 2)$

First find the slope of the line represented by $y + 3 = 2x$ by putting it in slope-intercept form. This is the Green Line.

$$\begin{array}{r} y + 3 = 2x \\ -3 \quad -3 \\ \hline y = 2x - 3 \rightarrow \text{The slope is 2.} \end{array}$$

Since the slope of the green line is 2 and the green line and blue line (that doesn't have an equation yet) are perpendicular, this means they have the slopes that are OPPOSITE RECIPROCALs, so the blue line has a slope of $-\frac{1}{2}$, and we know that it passes through the point $(-5, 2)$, so we now have enough information to replace m , x , and y , in:

$$\begin{array}{l} y = mx + b \\ 2 = -\frac{1}{2}(-5) + b \\ 2 = \frac{5}{2} + b \\ -\frac{5}{2} \quad -\frac{5}{2} \\ \hline -\frac{1}{2} = b \end{array} \quad \text{So the equation is:}$$
$$y = -\frac{1}{2}x - \frac{1}{2}$$

Ex: Determine which lines, if any, are parallel or perpendicular:

a. $y = \frac{3}{5}x + 1$

b. $5y = 3x - 2$

c. $10x - 6y = -4$

In order to decide if lines are parallel or perpendicular you need to know their slopes.

Since line a is already in slope-intercept form, the slope is:

$$m_a = \frac{3}{5}$$

Lines b and c must be put into slope-intercept form before identifying the slope:

$$\begin{aligned} \text{Line b: } \frac{5y}{5} &= \frac{3x-2}{5} \\ y &= \frac{3}{5}x - \frac{2}{5} \end{aligned}$$

$$m_b = \frac{3}{5}$$

$$\begin{aligned} \text{Line c: } 10x - 6y &= -4 \\ \frac{-10x}{-6} - \frac{-10x}{-6} & \\ \frac{-6y}{-6} &= \frac{-4 - 10x}{-6} \end{aligned}$$

$$y = \frac{2}{3} + \frac{5}{3}x$$

$$m_c = \frac{5}{3}$$

Since lines a and b both have the same slope they are parallel.

Line c's slope is the reciprocal, but not the opposite of a and b so none are perpendicular.

5.4: Write Equations of Lines in Standard Form

Write two equivalent equations in standard form:

Ex: $3x - 6y = 9$ you can multiply by any number you want (as long as you do it every item). Some possible answers are:

$$x - 2y = 3$$

$$6x - 12y = 18$$

$$9x - 18y = 27$$

Write equations of lines in standard form using the given information:

Ex: (4, 4) and (8, 2)

Ex: (-2, 3) and (-4, -5)

1) To write an equation in standard form you must first write it in slope-intercept form (which means you have to find the slope and the y-intercept)

$$m = \frac{2-4}{8-4} = \frac{-2}{4} = -\frac{1}{2}$$

$$4 = -\frac{1}{2}(4) + b$$

$$4 = -2 + b$$

$$6 = b$$

$$y = -\frac{1}{2}x + 6$$

$$m = \frac{-5-3}{-4-(-2)} = \frac{-8}{-2} = 4$$

$$3 = 4(-2) + b$$

$$3 = -8 + b$$

$$11 = b$$

$$y = 4x + 11$$

2) Once written in slope-intercept, move the x term over to join the y term to begin to get in standard form.

$$\begin{array}{r} y = -\frac{1}{2}x + 6 \\ + \frac{1}{2}x + \frac{1}{2}x \\ \hline 2(\frac{1}{2}x + y = 6) \end{array}$$

$$\begin{array}{r} y = 4x + 11 \\ -4x \quad -4x \\ \hline -1(-4x + y = 11) \end{array}$$

3) If needed, adjust so A, B, and C, are integers and A is positive.

The green portion above shows what you would need to multiply by in each problem to make A, B, and C meet the required criteria. The final answers would be:

$$x + 2y = 12$$

$$4x - y = -11$$

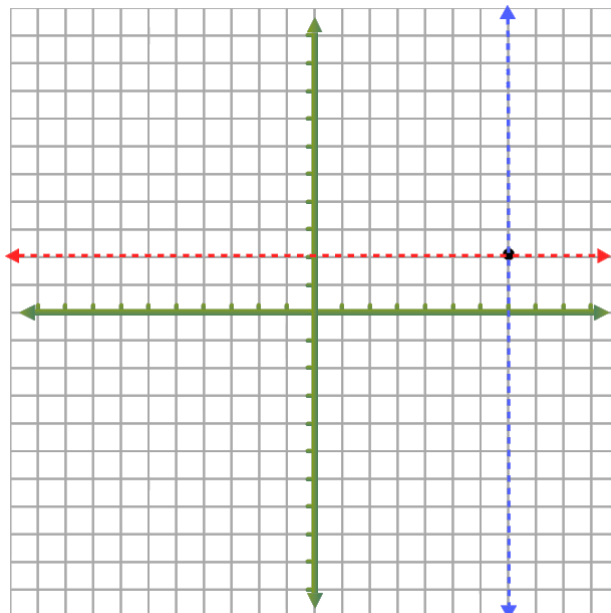
Ex: Write the equations of the horizontal and vertical lines that pass through the point (7, 2)

Vertical lines are made up of points that have all x's in common. In this case, on the blue line, the x-coordinate is always 7 which means the equation is:

$$x = 7$$

Horizontal lines are made up of points that have all y's in common. In this case, on the red line, the y-coordinate is always 2, which means the equation is:

$$y = 2$$



Complete equations in standard form:

Ex: $5x + By = 6$; (2, 1)

Ex: $Ax + 5y = 7$; (4, 3)

Replace x and y with the information from the given ordered pair.

$$\begin{aligned}5(2) + B(1) &= 6 \\10 + B &= 6 \\B &= -4\end{aligned}$$

$$\begin{aligned}A(4) + 5(3) &= 7 \\4A + 15 &= 7 \\4A &= -8 \\A &= -2\end{aligned}$$

Plug in the missing coefficient back into the original equation:

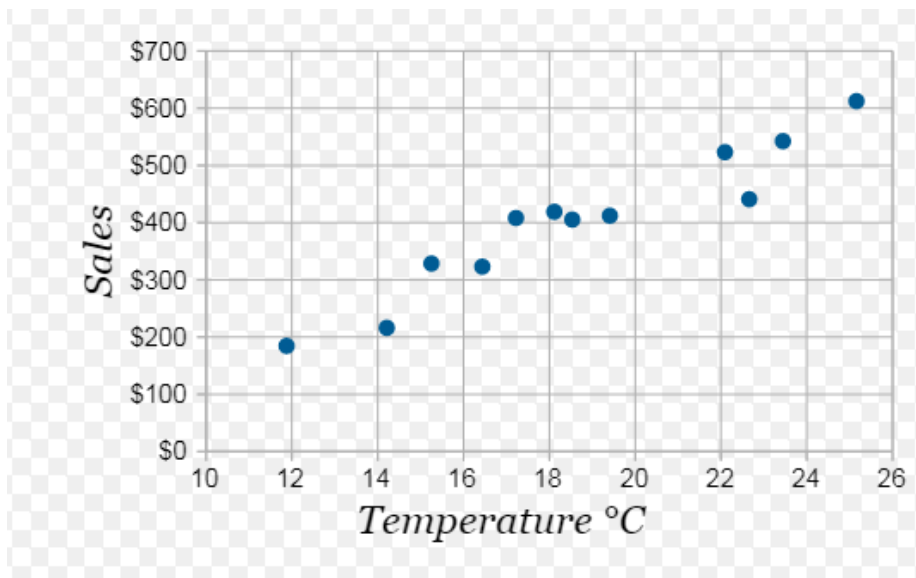
$$5x - 4y = 6$$

$$-2x + 5y = 7$$

Make sure A , B , and C meet the required criteria.

$$2x - 5y = -7$$

Use the scatter plot below to determine the type of association and decide which line is most representative of the relationship between x and y .



A. Please choose the best description of the relationship between x and y .

1. Relatively no correlation 2. Positive correlation 3. Negative correlation

It is a positive correlation

B. Which equation provides the best description of the relationship between x and y ?

1. $y = 35x$ 2. $y = -12x + 150$ 3. $y = 30x + 150$ 4. $y = 25x - 10$

Equation 3 ($y = 30x + 150$) because this slope and y -intercept appears to match. The others do not.